A Note on Common Trends and Convergence

by

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April, 2003

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Introduction

In the following note, we briefly review the convergence literature pointing the attention on the empirical issues. During the last decades, using different cross-sectional, time series, and panel empirical models, a growing number of studies have analysed if poor countries converge towards richer countries. Many recently supplied empirical models suffer from the strong hypotheses that the speed of convergence over time and across countries and the growth rate of technological progress across countries are homogeneous (see for example Barro and Sala-I-Martin,1995).

In the note, we show as different speed of convergence across countries may not influence convergence but homogeneity over time and across countries of the technological growth rate is required in order to rule out the possibility of divergence.

We follow three different approaches to assess for the presence of convergence or divergence of real per-capita output. All approaches are essentially based on common trend analysis.

The first methodology analyses if differences between log per capita income across countries are characterised by common and/or idiosyncratic stochastic trends. If this is the case, we will have evidence of divergence of per capita real income across countries.

The second approach filters the series of log per capita income for each country by extracting the Hodrick and Prescott (1997) trend-components. We analyse if these components converge monotonically across countries towards a common component. If this is the case we can say that poor countries will converge with richer countries.

Finally we use a third method, mainly based on least squares panel estimation, to infer if convergence can characterize labour productivity across countries and sectors.

1. Defining transition of log per-capita real income in neoclassical growth theory.

Assume that at each time output $Y(t)$ can be described by the following Cobb-Douglas production function

$$Y(t) = F\left(K(t), L(t)A(t)\right) = B\left(K(t)\right)^{a}\left(L(t)A(t)\right)^{1-a} \qquad 0<a<1 \qquad (1.1)$$

where $K(t), L(t)A(t), a$ are respectively the stock of net capital, the effective stock of labour as product of the amount of labour per labour efficiency, and the share of capital in total output. B is a multiplicative constant.

The net increase in the stock of capital at time t, $\dot{K}$, can be written as difference between the gross investment $I(t)$ and capital depreciation $d\,K(t)$

$$\dot{K} = \frac{dK}{dt} = I(t) - d\,K(t) = sF\left(K(t), L(t)A(t)\right) - d\,K(t) = s\left[B\left(K(t)\right)^{a}\left(L(t)A(t)\right)^{1-a}\right] - d\,K(t) \qquad (1.2)$$

where s is the constant saving rate.

The stock of capital and output can be expressed in terms of effective labour unit, i.e. $k(t) = K(t)/\left[L(t)A(t)\right]$, $y(t) = Y(t)/\left[L(t)A(t)\right]$. Thus equation (1.1) can be rewritten as

$$y(t) = F\left(k(t)\right) = B\left[k(t)\right]^{a} \tag{1.3}$$

and (1.2) as

$$\frac{\dot{K}}{\left[L(t)A(t)\right]} = sF\left(k(t)\right) - d\,k(t) = sBk(t)^{a} - d\,k(t) \tag{1.4}$$

Note that

$$\dot{k} = dk/dt = \frac{\dot{K}\ L(t)A(t) - \left(\dot{L}A(t) + \dot{A}L(t)\right)K(t)}{\left(L(t)A(t)\right)^{2}} = \frac{\dot{K}}{\left[L(t)A(t)\right]} - \left(n + x\right)k(t)$$

where n and x are respectively the exogenous growth rate of population and the exogenous technology growth rate. Substituting the previo us expression into (1.4), we end with

$$\dot{k} = sBk(t)^{a} - \left(n + d + x\right)k(t) \tag{1.5}$$

and

$$g_{k} = \dot{k}/k = sBk(t)^{a-1} - \left(n + d + x\right) \tag{1.6}$$

From (1.6) we can infer the first important result

$$dg_{k}/dk = \left(a - 1\right)sBk(t)^{a-2} < 0, \tag{1.7}$$

when the per-capita stock of capital rise its growth rate decrease. In other term, the higher is the per-capita stock of capital the lower is its growth rate.

Now, we will try to solve the differential equation (1.5). Rewrite the equation as

$$\dot{k}k(t)^{-a} + \left(n + d + x\right)k(t)^{1-a} = sB \tag{1.8}$$

and define $v(t) = k(t)^{1-a}$ so as $dv/dt = \left(1 - a\right)\left(dk/dt\right)k(t)^{-a}$. Substituting into (1.8)

$$\frac{dv}{dt}\frac{1}{\left(1-a\right)} + \left(n + d + x\right)v(t) = sB \tag{1.9}$$

which is a simple first order linear autonomous differential equation in $v(t)$. The particular integral is found writing $v(t) = \overline{c}$, from which $dv/dt = 0$. The solution is $\overline{c} = sB/\left(n + d + x\right)$. The complementary function can be simply found by writing $v(t) = Me^{rt}$ and $dv/dt = rMe^{rt}$ so that we end with

$$rMe^{rt}\frac{1}{(1-a)}+(n+d+x)Me^{rt}=0$$

and finally writing $r=-(1-a)(n+d+x)$, the solution of (1.9) is given by:

$$v(t)=k(t)^{1-a}=\frac{sB}{(n+d+x)}+Me^{-(1-a)(n+d+x)t} \tag{1.10}$$

The convergence literature has shortly labelled the coefficient $(1-a)(n+d+x)$ as *beta coefficient*, and we will follow this convention writing $b=(1-a)(n+d+x)$.

Now the only thing that we have to do is defining M in (1.10), i.e we have to find the exact solution. At time zero, $M=\left\{\left[k(0)\right]^{1-a}-\frac{sB}{(n+d+x)}\right\}$ and we can finally write

$$k^{1-a}=\frac{sB}{(n+d+x)}+\left\{\left[k(0)\right]^{1-a}-\frac{sB}{(n+d+x)}\right\}e^{-bt} \tag{1.11}$$

Note first that for $t\to\infty$, the steady-state solution $\hat{k}$ is equal to

$$\hat{k}=\left[\frac{sB}{(n+d+x)}\right]^{\frac{1}{1-a}} \tag{1.12}$$

Second, the gap between the initial value and the steady-state value vanishes exactly at the constant rate $b=(1-a)(n+d+x)$.

This property can appreciated by looking at the following expression. It is simple to show that the following relationship holds

$$g_k\simeq-b\log\left(\frac{k(t)}{\hat{k}}\right) \tag{1.13}$$

i.e. the stock of capital per effective unit of labour approaches to steady-state at the constant rate b , where g_k has been previously defined in (1.6).

Proof:

From (1.6) we write k as

$$k(t)=\left[\frac{g_k+(n+d+x)}{sB}\right]^{-\frac{1}{1-a}}$$

and dividing both sides by (1.12) we have

$$\frac{k(t)}{\hat{k}} = \left[\frac{g_k + (n+d+x)}{sB} \frac{sB}{(n+d+x)} \right]^{-\frac{1}{1-a}}$$

and simplifying

$$\log\left(\frac{k(t)}{\hat{k}}\right) = -\frac{1}{1-a}\log\left(1+\frac{g_k}{(n+d+x)}\right) \simeq -\frac{1}{1-a}\frac{g_k}{(n+d+x)}$$

From the previous expression, we obtain immediately (1.13)

$$g_k \simeq -(1-a)(n+d+x)\log\left(\frac{k(t)}{\hat{k}}\right) \simeq -b\log\left(\frac{k(t)}{\hat{k}}\right). \qquad €$$

2. Output dynamics.

Note that from (1.3) and (1.13) the following relationship holds

$$g_y = ag_k \simeq -a\ b\log\left(\frac{k(t)}{\hat{k}}\right)$$

or

$$g_y \simeq -a\ b\log\left[\frac{(y(t)/B)^{1/a}}{(\hat{y}/B)^{1/a}}\right] = -b\log\left[\frac{y(t)}{\hat{y}}\right] \tag{2.1}$$

Equation (2.1) can also be expressed as

$$\frac{d\log y(t)}{dt} + b\log y(t) = b\log\hat{y} \tag{2.2}$$

which is a linear differential equation in $\log y(t)$. The particular integral is given by defining $\log y(t) = \overline{c}$, from which $d\log y(t)/dt = 0$. The solution is $\overline{c} = \log\hat{y}$. The complementary function can be find writing $\log y(t) = Me^{rt}$ and $d\log y(t)/dt = rMe^{rt}$ and we end with the following relationship $rMe^{rt} + bMe^{rt} = 0$. Here the solution is $r = -b$.

Thus the solution for (2.2) is given by

$$\log y(t) = \log\hat{y} + Me^{-bt} \tag{2.3}$$

For t=0, $M = [\log y_0 - \log\hat{y}]$, so we end with

$$\log y(t) = \log\hat{y} + [\log y(0) - \log\hat{y}]e^{-bt} \tag{2.4}$$

Finally we can rewrite (2.4) in term of per-capita income taking into account that $y(t) = Y(t)/[L(t)A(t)] = \tilde{y}(t)A(t)$ where $\tilde{y}(t) = Y(t)/L(t)$. Substituting into (2.4)

$$\log\tilde{y}(t) = \log\hat{y} + [\log y(0) - \log\hat{y}]e^{-bt} + \log A(t) \tag{2.5}$$

If we assume that technological progress evolves as $A(t) = A(0)e^{xt}$, equation (2.5) can be finally expressed as

$$\log \tilde{y}(t) = \log \hat{y} + [\log y(0) - \log \hat{y}]e^{-bt} + \log A(0) + xt. \tag{2.6}$$

3. Convergence

Equation (2.6) can be used to compute the rate of growth of per-capita real income for any country *i*

$$\log\left(\frac{\tilde{y}_i(t)}{\tilde{y}_i(t-1)}\right) = [\log y_i(0) - \log \hat{y}_i](1 - e^{b_i})e^{-b_i t} + x_i \tag{3.1}$$

and for $t \to \infty$

$$\log\left(\frac{\tilde{y}_i(t)}{\tilde{y}_i(t-1)}\right) = x_i \tag{3.2}$$

i.e in the long-run the rate of growth of per-capita income in any economy converges to the exogenous rate of growth of technological progress. This result of the neoclassical growth model has important consequence on convergence between per capita income of any pair of countries *i* and *j*.

Using (3.2) and it is simple to see that

$$\log\left(\frac{\tilde{y}_i(t)}{\tilde{y}_i(t-1)}\right) - \log\left(\frac{\tilde{y}_j(t)}{\tilde{y}_j(t-1)}\right) = x_i - x_j \tag{3.3}$$

Thus, if two countries experience different rates of growth of technological progress their incomes <u>cannot</u> converge. In other term, convergence requires that $x_i = x_j = x$.

Assume now that $x_i = x_j$ and the convergence rate $b = (1-a)(n+d+x) > 0$ is identical between any pair of countries and they converge to the same steady-state $\hat{y}_i = \hat{y}_j = \hat{y}^*$, this is the homogeneous hypothesis stated in Barro and Sala-I-Martin (1995). The left side of equation (3.3) now will be equal to

$$\log\left(\frac{\tilde{y}_i(t)}{\tilde{y}_i(t-1)}\right) - \log\left(\frac{\tilde{y}_j(t)}{\tilde{y}_j(t-1)}\right) = [\log y_i(0) - \log y_j(0)](1 - e^{b})e^{-bt} \tag{3.4}$$

From (3.4) we note first that as $t \to \infty$

$$\log\left(\frac{\tilde{y}_i(t)}{\tilde{y}_i(t-1)}\right) - \log\left(\frac{\tilde{y}_j(t)}{\tilde{y}_j(t-1)}\right) = 0,$$

i.e the two growth rates converge in the long-run. During the transition period

$$\log\left(\frac{\tilde{y}_i(t)}{\tilde{y}_i(t-1)}\right)-\log\left(\frac{\tilde{y}_j(t)}{\tilde{y}_j(t-1)}\right)<0 \quad \Leftrightarrow \log y_i(0)>\log y_j(0). \tag{3.5}$$

Relationship (3.5) show that the country that has the lower level of per-capita income in the starting period will grow at a higher rate during the transition period. This is a well known property of neoclassical growth models and a full discussion is contained in Barro and Sala-I-Martin (1995) when they analyse the absolute converge hypothesis.

Assume now that $b_i \neq b_j$, the differences between the growth rates can be rewritten as

$$\log\left(\frac{\tilde{y}_i(t)}{\tilde{y}_i(t-1)}\right)-\log\left(\frac{\tilde{y}_j(t)}{\tilde{y}_j(t-1)}\right)=\left[\log y_i(0)-\log \hat{y}^*\right]\left(1-e^{b_i}\right)e^{-b_i t}-\left[\log y_j(0)-\log \hat{y}^*\right]\left(1-e^{b_j}\right)e^{-b_j t}.$$

As before, when $t \to \infty$ the two growth rates converge. During transition period, it is useful to analyse the sign of $\log\left(\tilde{y}_{it}/\tilde{y}_{it-1}\right)-\log\left(\tilde{y}_{jt}/\tilde{y}_{jt-1}\right)$. Assume that the following relationship holds: $\log \hat{y}^* > \log y_i(0) > \log y_j(0)$, i.e both countries converge to the same steady-state per-capita income and the initial level of income of country i is higher than the initial income of country j. Country j will grow at a higher rate than country i when

$$\left[\log y_i(0)-\log \hat{y}^*\right]\left(1-e^{b_i}\right)e^{-b_i t}-\left[\log y_j(0)-\log \hat{y}^*\right]\left(1-e^{b_j}\right)e^{-b_j t}<0$$

or

$$\frac{\left[\log y_i(0)-\log \hat{y}^*\right]}{\left[\log y_j(0)-\log \hat{y}^*\right]}\frac{\left(1-e^{b_i}\right)}{\left(1-e^{b_j}\right)}e^{-\left(b_i-b_j\right)t}<1 \tag{3.6}$$

The first term in square bracket of (3.6) is by definition less than unity. If $b_i > b_j$ the second factor is higher than unity but the third factor is less than unity. We have from (3.6) that when t is small, the growth rate of an initially poor country may be lower than the growth rate of an initially rich country and divergence may occur during the early period of transition, but converge can hold in the long run. This story can be shown in the following figure.

Country A is the richest country. It has a higher starting per-capita income relatively to countries B, i.e $\log\left(y_A(0)\right)>\log\left(y_B(0)\right)$. If $b_A > b_B$, in the initial period per-capita income can diverge but ultimate convergence holds in the long run. Thus we can have convergence in the presence of different rates of convergence.

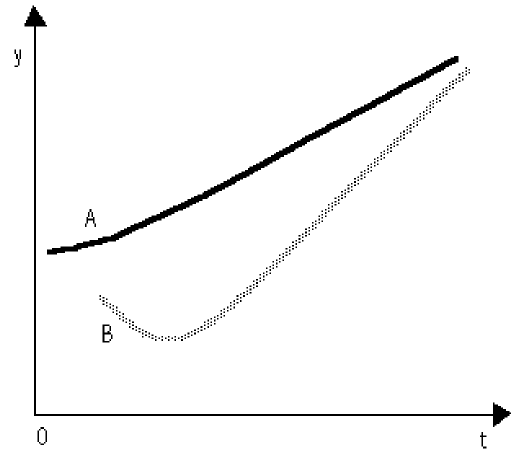

Fig. 1 Transitional Convergence

4. How many years for convergence?

Equation (2.5) can be used to compute the time value t^* for which per-capita income of poorer country may reach k percent of per-capita income of richer country. Following Figure 1, assume that A is the rich country and B is the poor country, thus

$$\log\left(\frac{y_B\left(t^*\right)}{y_A\left(t^*\right)}\right) = \left[\log y_B(0) - \log \hat{y}\right] e^{-b_B t^*} - \left[\log y_A(0) - \log \hat{y}\right] e^{-b_A t^*} = \log k \qquad (4.1)$$

For $y_B(0) = €400$, $y_A(0) = €8000$, $\hat{y} = € \, 15000$, $b_B = 0.005$, $b_A = 0.03$, and finally k =0.9, i.e. 90%. It is simple to find that in this case $t^* = 541$, i.e. 541 years occur to country B to catch up 90% of per-capita income of richer country A.

5. Econometric specification

Following Phillips and Sul (2003), first we will simplify equation (19), extracting a common trend component which possibly could be shared by all the countries. After that, we will introduce a idiosyncratic component, which absorb country specific shocks. We denote $y_i(t)$ as y_{it}, in keeping with discrete time modelling convention. Using equation (2.6) the transition path of log of per-capita income can be written as

$$\begin{aligned} \log \tilde{y}_{it} &= \left[\log y_{i0} - \log \hat{y}\right] e^{-b_i t} + \log \hat{y} + \log A_t \\ &= h_i c_{it} + f_t = \left(1 + \frac{h_i c_{it}}{f_t}\right) f_t = l_{it} f_t \end{aligned} \qquad (5.1)$$

where $h_i = [\log y_{i0} - \log \hat{y}], c_{it} = e^{-b_i t}, f_t = \log \hat{y} + \log A_t$. Thus, during the transition period, l_{it} is function of the speed of convergence b_i, the initial per-capita income y_{i0} and the growth rate of technological progress x. Consider now the following empirical model

$$\log \tilde{y}_{it} = l_{it} f_t + e_{it} \qquad e_{it} = r e_{it-1} + u_{it} \tag{5.2}$$

where $u_{it} \equiv iid(0, s_u^2)$ and the error e_{it} can be thought as an idiosyncratic country-specific shock which influences per-capita income. [1]

Using the previous condition we are able to analyse convergence by comparing any pair of differences between log per-capita income of two countries or the difference between a country log per-capita income and its cross-section average across the N countries. In this case, if the previous conditions hold and using panel data it is simple to show that growth convergence will hold for $i = 1, 2, \ldots, N$ when

$$\log \tilde{y}_{it} - \log \tilde{y}_{jt} = (l_{it} - l_{jt}) f_t + (e_{it} - e_{jt}) = I(0) \text{ for } all\ i \text{ and } j \tag{5.3}$$

and

$$\log \tilde{y}_{it} - \frac{1}{N}\sum_{i=1}^{N} \log \tilde{y}_{it} = (l_{it} - \bar{l}) f_t + (e_{it} - \bar{e}_t) = I(0) \text{ for } all\ i \text{ and } j. \tag{5.4}$$

where $\bar{l}_t = \frac{1}{N}\sum_{j=1}^{N} l_{jt}$ and $\bar{e}_t = \frac{1}{N}\sum_{j=1}^{N} e_{jt}$.

Thus, convergence requires that for any pair of countries the log per-capita income difference must be a stationary panel and equations (5.3) and (5.4) highlight why panel unit root tests have been largely used to address the convergence issue.

Over the last few years, a lot of attention has been paid to the nonstationary property of panels. As is well known, many studies have examined whether the time series behavior of economic variables is consistent with a unit root (see for a survey Diebold and Nerlove, 1990; Campbell and Perron 1991). In general, the analysis has been carried out using tests such as the augmented Dickey-Fuller's (ADF) (Dickey and Fuller, 1981) test or semi-parametric tests, as in the case of the Phillips-Perron tests (Phillips and Perron, 1988). The main problem here is that, in a finite sample, any unit roots process can be approximated by a trend-stationary process. For example, the simple difference stationary process $y_t = f y_{t-1} + e_t$ with $f = 1$ can be arbitrarily

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[1] It is straightforward to augment (5.1) including a specific growth rate of technological progress x_i. In this case, as required by Solow's neoclassical growth model, as t grows large convergence requires that $x_i \to x$.

well approximated by a stationary process with f less than but close to one. The result is that unit root test statistics have limited power against the alternative.

Recently, starting from the seminal works of Quah (1990, 1994), Breitung and Meyer (1991) Levin and Lin (1992, 1993), Im *et al.* (1997) many tests have been proposed which attempt to introduce unit root tests in panel data. They show that combining the time series information with that from the cross-section, the inference about the existence of unit roots can be made more straightforward and precise, especially when the time series dimension of the data is not very long, and similar data may be obtained across a cross-section of units such as countries or industries. In any case, all the panel unit root tests suffer from the serious limitation that the cross-sectional units are uncorrelated. This means for example hypothesizing that European countries' labour productivities are not correlated either in the short or long term, where it seems clear that high cross-correlations exist and are relevant.

Three papers which have been presented in recent years, Bai and Ng (2003), Moon and Perron (2002) and Phillips and Sul (2002,) take this problem into account. In brief, each of these proposes a dynamic factor model in which the panel data is generated by one or more factors that are common to all the individual units (but which may exert different effects on the individual unit) and by idiosyncratic shocks that are uncorrelated across all the individual units. While Moon and Perron (2002) and Phillips and Sul (2002) state that common factor(s) must be a stationary variable(s), Bai and Ng (2003) include the knowledge obtained from previous works which permit nonstationary (or stationary) common component(s). For this reason we are concentrating our attention on Bai and Ng's (2003) model.

Assuming that in each sector *i*, the logarithm of per-capita income can be decomposed as

$$\log \tilde{y}_{it} - \log \overline{y}_t = c_{it} + \left(l_{it} - \overline{l} \right)' f_t + \left(e_{it} - \overline{e}_t \right) \quad i = 1,\ldots,N \quad t = 1,\ldots,T \qquad (5.5.1)$$

$$\left(I - L \right) f_t = C\left(L \right) v_t \qquad (5.5.2)$$

$$\left(1 - r_i L \right)\left(e_{it} - \overline{e}_t \right) = B_i\left(L \right) e_{it} \qquad (5.5.3)$$

where $\log \overline{y}_t = \frac{1}{N}\sum_{i=1}^{N} \log \tilde{y}_{it}$; $\overline{l}$, $\overline{e}_t$ has been defined in (5.4), c_{it} is an individual deterministic constant or linear trend, f_t is a $\left(r \times 1 \right)$ constant, when $r = 1$, or vector, when $r > 1$, of common factor(s) and $\left(l_i - \overline{l} \right)$ is the corresponding vector of factor loadings which, contrary to equation (5.1), does not depend on time *t*. The error terms n_t and e_{it} are mutually independent across *i* and *t*, and $B_j\left(L \right)$ and $C\left(L \right)$ are two polynomial, with a rank of $C\left(1 \right) = r_1$. In synthesis, when

$r_1 = 0$, $C(1) = 0$, and (5.5.2) is over-differenced, while for $r_1 \geq 1$ the system contains one or more common stochastic trends. Note from (5.5.3), that the idiosyncratic term $\left(e_{jt} - \overline{e}_t\right)$ is stationary when $|r_i| < 1$ and non-stationary, or equivalently, integrated of order one $I(1)$, for $r_i = 1$.

In brief, Bai and Ng's (2003) model consists in estimating common factor(s), f_t, and idiosyncratic components by applying the method of principal components to the first differenced data $\Delta\mathbf{log}\tilde{\mathbf{y}}$ (where now $\Delta\mathbf{log}\tilde{\mathbf{y}}$ is the observed $(T \times N)$ matrix of (demeaned) differenced log per-capita income for the N countries and over T periods), and obtaining the (differenced) common factor(s) as the first r_1 eigenvectors with the largest eigenvalues of the matrix $\Delta\mathbf{log}\tilde{\mathbf{y}}\Delta\mathbf{log}\tilde{\mathbf{y}}'$. Factor loading l_i can be easily calculated as the product of (transposed) $\Delta\mathbf{log}\tilde{\mathbf{y}}$ matrix and common factor(s). Thus, the (differenced) idiosyncratic terms in (5.5.1) can be calculated as $\left(\Delta \log \tilde{y}_{it} - \Delta \log \overline{y}_t\right) - \hat{l}_i' \Delta \hat{f}_t = \left(\Delta\hat{e}_{it} \text{-} \Delta\overline{e}_t\right)$. Finally, the estimate of the level of common factor(s) can be obtained simply by integrating $\hat{f}_t = \sum_{k=2}^{T} \Delta\hat{f}_k$, the idiosyncratic error term can be computed as $\left(\hat{e}_{it} - \overline{e}\right)_t = \sum_{k=2}^{T} \left(\Delta\hat{e}_{ik} - \Delta\overline{e}_k\right)$.

The decomposition between common and idiosyncratic effects can be used to value the importance of the two components in determining the dynamics of per-capita income. Moreover in this case we can use both the estimated common factor(s) $\hat{f}_t$ and error term $\left(\hat{e}_{it} - \overline{e}_t\right)$ to analyse the stochastic properties of both components.

5.1 Empirical Analysis

We will apply the analysis to the real per-capita GDP variable (labelled RGDPCH) collected by Heaston et al. (2002). Our dataset includes 101 countries and the variable is spanned during the period 1960-1999.

The first task when computing multifactor analysis as in (5.5.1), is to specify correctly the number of factors *r*. We follow Bai and Ng (2002) and we use what they label BIC_3 criterion given by

$$BIC_3(r) = \min_r \frac{1}{NT} \sum_{j=1}^{N} \sum_{t=1}^{T} \left(X_{jt} - l_j^{r'} \hat{f}_t^r \right)^2 + r\hat{s}^2 \left(\frac{(N+T-r)\ln(NT)}{NT} \right) \quad (5.6)$$

where $\hat{s}^2 = 1/(NT)\sum_{j=1}^{N}\sum_{t=1}^{T} E(e_{jt})^2$. [2] The number of factors $\hat{r}$ are specified such that $\hat{r} = \underset{0\le r\le r\max}{\arg\min}\, BIC_3(r)$. Thus, the criteria defines the correct number of factors taking into account of the mean squared sum of residuals, i.e the first addend in (5.6), plus a penalty function for over-fitting given by the second term in (5.6). Bai and Ng (2002) show that this criterion performs well for our sample size of data.

We compute (5.6) using as maximum number of factors $r\max = 12$. The BIC_3 criterion suggests the presence of a single common factor. Thus, we estimate (5.5.1-5.5.3) including only one factor, i.e. the common factor $\hat{f}$ is given by $\sqrt{T-1}$ times the eigenvector of $\Delta\mathbf{log}\tilde{\mathbf{y}}\Delta\mathbf{log}\tilde{\mathbf{y}}'$ with the largest eigenvalue and the estimated factor loading vector $\hat{l} = \Delta\mathbf{log}\tilde{\mathbf{y}}'\hat{f}/(T-1)$.

It is simple now to derive what we call common factor component. Note that for each country *i* the estimated common component can be expressed as

$$(1-L)(\log\tilde{y}_{it} - \log\overline{y}_t) = (\hat{l}_i - \overline{l})(1-L)\hat{f}_t$$

where L is the lag operator such that $Ly_t = y_{t-1}$. The previous expression can be easily integrated

$$(\log\tilde{y}_{it} - \log\overline{y}_t) = (\log\tilde{y}_{i0} - \log\overline{y}_0) + \hat{l}_i\hat{f}_t$$

and taking exponential from both sides

$$\tilde{y}_{it} = \tilde{y}_{i0}\left(e^{\log\overline{y}_t - \log\overline{y}_0}\right)e^{(\hat{l}_i - \overline{l})\hat{f}_t}. \qquad (5.7)$$

Finally, we can calculate the cross-countries average of (5.7) and figure 1 shows the results.

In table 1 we provide test statistics on unit root and autoregressive coefficient estimates. Concentrating attention on ADF test for common factor component, four lags of differenced variable have been included, we note that it does not reject the null hypothesis of a unit root in the common component. The value of the test is –2.68 (5% asymptotic critical value is –2.86). The same results we obtain when using Kwiatlowski et al. (1992) (KPSS) test where now the null is stationarity. The statistic is 3.091 (5% asymptotic critical value is 0.46) well above the 5% critical value. Thus using both ADF and KPSS tests we conclude for common factor component has a unit root.

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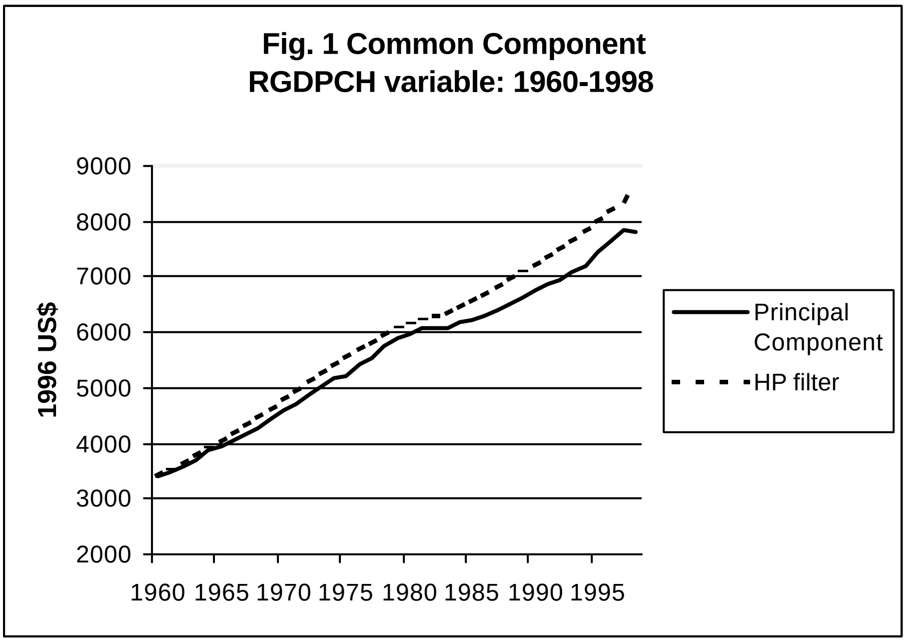
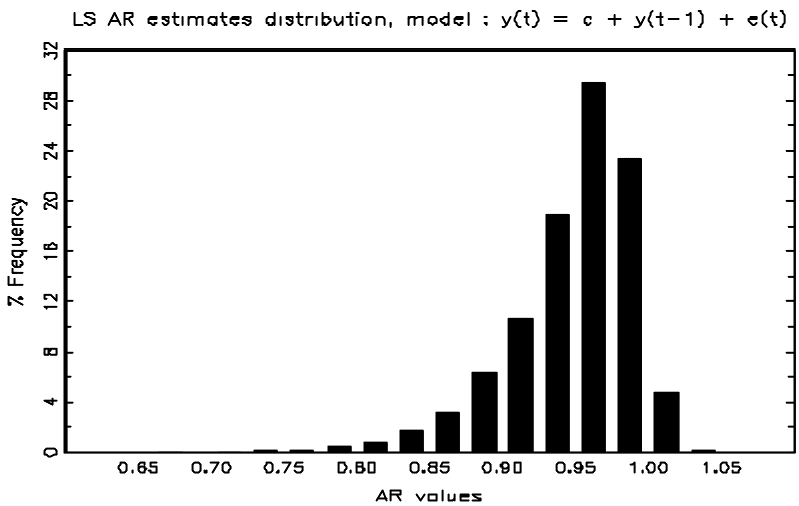
[2] In our case $X_{jt} = \log\tilde{y}_{it} - \log\overline{y}_t$.



We can now test for unit roots on idiosyncratic components. Following Maddala and Wu (1999), we pool ADF and KPSS tests and find that both test reject the null of unit root. Thus, we conclude that the idiosyncratic components are stationary variables.

In the second section of the table we report the autoregressive estimates of common factor component and idiosyncratic component. As is well known, the AR parameter estimated by least squares (LS) tends to be downward biased and the bias is quite large as the AR parameter gets close to 1.

The following graph shows LS distribution when the AR(1) process has length T=100 and $r = 1$ and 10.000 replications have been computed.



From the graph we infer that LS estimator is not median-unbiased, i.e. the LS estimator of the AR models with unit roots has asymmetric distribution.

Table 1. Some statistics on Unit Roots: common factor component and idiosyncratic components

	Common Factor	Idiosyncratic component
Unit root tests:		
ADF test	-2.68	-
KPSS test	3.091	-
Pooled ADF	-	4.946
Pooled KPSS	-	94.160
Autoregressive estimates:		
OLS	0.974	-
Median (Andrew, 1993)	1.000	-
Average OLS	-	0.933
Minimum OLS	-	0.663
Maximum OLS	-	1.044
Average Median	-	0.947
Minimum Median	-	0.680
Maximum Media	-	1.000
n°. unit root (median estimates)	-	23

Thus, when the parameter space is bounded or when the distributions of estimators are skewed and/or kurtotic, the concept of median-unbiasedness is often more useful than that of mean-unbiasedness.

Andrew (1993) proposed a median-unbiased estimators of the AR parameter. The median estimator may be a preferred measure to the mean because the median is less sensitive to the tails of the distribution. In table 1 we report the value of LS estimates and Andrew's (1993) median estimates for the common factor component and idiosyncratic components.

First, note that the LS autoregressive estimate of the common factor component is 0.974 while median estimate is 1.0. Thus, when using Andrew's (1993) methodology, we find that the autoregressive parameter is unity and this result is in line with the previously commented unit root test statistics. When analysing idiosyncratic components, we find that on average the AR estimates is lower than 1, but for 23 countries out of 101 the estimated median is unity, thus highlighting a unit root in the autoregressive process.

Concluding, first the results show that when analysing the difference between the log of per capita income for wide sample of countries and their cross-section average, the common factor component has a unit root or, using a well known definition, has a stochastic trend while, on average, the idiosyncratic component is a stationary variable. Second, if we use a stationary trend to highlight technological progress, we use a poor specification. Technological trend cannot be represented by a stationary trend and a stochastic trend is more representative. Third, idiosyncratic shocks are stationary variables, i.e. their effects tend to disappear during the time. Fourth, convergence of per-capita income does not hold. Unless $\left(l_i - \overline{l}\right) = 0$, i.e we have a singular distribution of factor loadings the proposition stated in (5.4) does not hold.

6. Hodrick and Prescott (1997) filter.

The conceptual framework of Hodrick and Prescott (1997) (HP hereafter) filter relies on the hypothesis that the logarithm of a given time series $\log y_t$ is the sum of a growth component g_t and cyclical component c_t

$$\log y_t = g_t + c_t \qquad t = 1,\dots,T \tag{6.1}$$

HP define first the smoothness of growth component path as the sum of the squares of its second difference and second they assume that over long time periods the average of cyclical component is near zero. Note as the trend component g_t can be associated to the term $l_{it} f_t$ in (5.2).

The previous HP hypotheses lead to a programming problem for determining the growth components g_t

$$L = \min_{\{g_t\}_{t=-1}^{T}} \left\{ \sum_{t=1}^{T} c_t^2 + l \sum_{t=1}^{T} \left[\left(g_t - g_{t-1}\right) - \left(g_{t-1} - g_{t-2}\right) \right]^2 \right\} \tag{6.2}$$

The Lagrange multiplier l is a positive number which penalises variability in the growth component series. The larger is the value of l, the smoother is the solution series. For a sufficiently large l, at the optimum all the $g_{t+1} - g_t$ must be arbitrarily near some constant b and therefore the g_t arbitrarily near the deterministic trend $g_0 + bt$, i.e. the limit of the solution to program (6.2) as l approaches infinity is the least squares fit of a linear time trend.

6.1 A straightforward solution

Equation (6.2) can be written as

$$L = \min_{\{g_t\}_{t=-1}^{T}} \Big\{ \ldots + (y_t - g_t)^2 + \ldots + l \Big[\ldots + (g_t - 2g_{t-1} + g_{t-2})^2 +$$

$$+ (g_{t+1} - 2g_t + g_{t-1})^2 + (g_{t+2} - 2g_{t+1} + g_t)^2 \Big]\Big\},$$

and as it is minimized over $\{g_t\}_{t=-1}^{T}$ the first order condition with respect to g_t is

$$\frac{dL}{dg_t} = -2(\log y - g_t) + l \left[2(g_t - 2g_{t-1} + g_{t-2}) - 4(g_{t+1} - 2g_t + g_{t-1}) + 2(g_{t+2} - 2g_{t+1} + g_t) \right] = 0,$$

and the following relationship holds

$$\log y_t = g_t + l \left[g_{t-2} - 4g_{t-1} + 6g_t - 4g_{t+1} + g_{t+2} \right].$$

From the previous equation, we have T linear first order equations, but the unknowns trends are T+4. We have to define the trends for $g_{-1}, g_0, g_{t+1}, g_{t+2}$.

One of the simplest hypothesis is defining the growth rate for these periods using the hypothesis that trend does not change quickly. Accordingly, the growth rate for $t \in \{-1, 0, t+1, t+2\}$ will be defined has $g_{t+1} - g_t = g_t - g_{t-1}$, and rearranging the four remaining equations are given

$$\begin{aligned}
g_0 &= 2g_1 - g_2 \\
g_{-1} &= 2g_0 - g_1 = 2(2g_1 - g_2) - g_1 = 3g_1 - 2g_2 \\
g_{T+1} &= 2g_T - g_{T-1} \\
g_{T+2} &= 2g_{T+1} - g_T = 2(2g_T - g_{T-1}) - g_T = 3g_T - 2g_{T-1}
\end{aligned}$$

We can now use the previous relationships, inserting them into the first order conditions, and solve the system for $\{g_t\}_{t=-1}^{T}$,

$$\begin{aligned}
\log y_1 &= g_1 + l \left[g_{-1} - 4g_0 + 6g_1 - 4g_2 + g_3 \right] = \\
&= g_1 + l \left[3g_1 - 2g_2 - 4(2g_1 - g_2) + 6g_1 - 4g_2 + g_3 \right] = \\
&= g_1 + l \left[g_{-1} - 2g_2 + g_3 \right], \\
\log y_2 &= g_2 + l \left[g_0 - 4g_1 + 6g_2 - 4g_3 + g_4 \right] = \\
&= g_2 + l \left[2g_1 - g_2 - 4g_1 + 6g_2 - 4g_3 + g_4 \right] = \\
&= g_2 + l \left[-2g_{-1} + 5g_2 - 4g_3 + g_4 \right], \\
\log y_3 &= g_3 + l \left[g_1 - 4g_2 + 6g_3 - 4g_4 + g_5 \right], \\
&\vdots
\end{aligned}$$

$$
\begin{aligned}
\log y_t \quad &= g_t + l\,[\,g_{t-2} - 4g_{t-1} + 6g_t - 4g_{t+1} + g_{t+2}\,],\\
&\ \vdots\\
\log y_{T-2} &= g_{T-2} + l\,[\,g_{T-4} - 4g_{T-3} + 6g_{T-2} - 4g_{T-1} + g_T\,],\\
\log y_{T-1} &= g_{T-1} + l\,[\,g_{T-3} - 4g_{T-2} + 6g_{T-1} - 4g_T + g_{T+1}\,] =\\
&= g_{T-1} + l\,[\,g_{T-3} - 4g_{T-2} + 6g_{T-1} - 4g_T + 2g_T - g_{T-1}\,] =\\
&= g_{T-1} + l\,[\,g_{T-3} - 4g_{T-2} + 5g_{T-1} - 2g_T\,],\\
\log y_T &= g_T + l\,[\,g_{T-2} - 4g_{T-1} + 6g_T - 4g_{T+1} + g_{T+2}\,] =\\
&= g_T + l\,[\,g_{T-2} - 4g_{T-1} + 6g_T - 4g_{T+1} + g_{T+2}\,] =\\
&= g_T + l\left[g_{T-2} - 4g_{T-1} + 6g_T - 4\left(2g_T - g_{T-1}\right) + 3g_T - 2g_{T-1}\right] =\\
&= g_T + l\,[\,g_{T-2} - 2g_{T-1} + g_T\,].
\end{aligned}
$$

Using matrix notation the previous first order conditions can be compactly written as

$$\log y = g + l\,D_{TxT}\,g = \left(I_{TxT} + l\,D_{TxT}\right)g \tag{6.3}$$

where $\log y = [\log y_1, \log y_2, \ldots, \log y_T]'$, $g = [g_1, g_2, \ldots, g_T]'$ and

$$
D_{TxT} = \begin{bmatrix}
1 & -2 & 1 & 0 & & & & & & \cdots & 0\\
-2 & 5 & -4 & 1 & 0 & & & & & & \vdots\\
1 & -4 & 6 & -4 & 1 & 0 & & & & & \\
0 & 1 & -4 & 6 & -4 & 1 & 0 & & & & \\
 & \ddots & & & & & & \ddots & & & \\
 & & 0 & 1 & -4 & 6 & -4 & 1 & 0 & & \\
 & & & \ddots & & & & & & \ddots & \\
 & & & & 0 & 1 & -4 & 6 & -4 & 1 & 0\\
 & & & & & 0 & 1 & -4 & 6 & 4 & 1\\
\vdots & & & & & & 0 & 1 & -4 & 5 & -2\\
0 & \cdots & & & & & & 0 & 1 & -2 & 1
\end{bmatrix}.
$$

Given l, the HP trend can be easily computed as

$$g = \left(I_{TxT} + l\,D_{TxT}\right)^{-1} \log y. \tag{6.4}$$

HP suggest a value of $l = 100$ for annual series (but Ravn and Uhlig (2002) have recently proposed a value of 6.25), and a value of $l = 1600$ for quarterly series. In figure 1 are shown the cross-section average of HP filters for the 101 countries in our sample and compare it with the previous estimate of common factor component.

The common component estimated via principal component shows a lower rate of growth than HP common component (a value of $l = 100$ has been used but the results do not change when adopting a value of $l = 6.25$).

We can use HP filter to analyse growth convergence/divergence process. The following statistics can be computed

$$\hat{h}_{it} = \frac{\hat{g}_{it}}{\frac{1}{N}\sum_{i=1}^{N}\hat{g}_{it}}$$

and using (5.2)

$$\hat{h}_{it} = \frac{\hat{l}_{it}\hat{f}_t}{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{l}_{it}\hat{f}\right)} = \frac{\hat{l}_{it}}{\frac{1}{N}\sum_{i=1}^{N}\hat{l}_{it}}$$

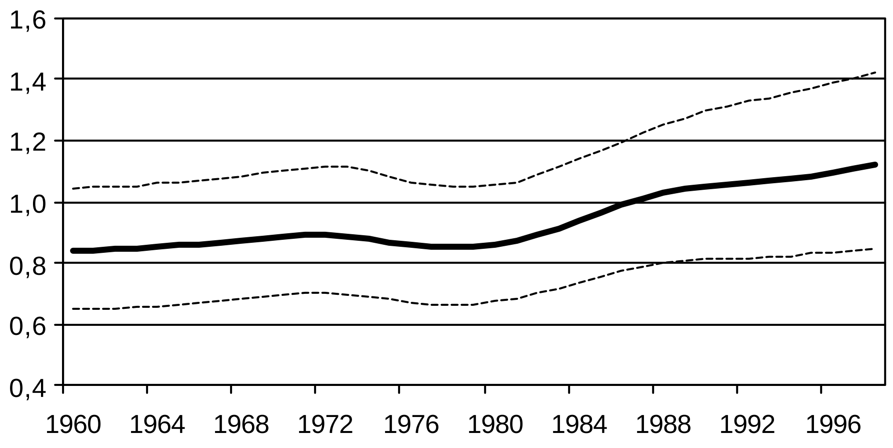
It is easy to see that under growth convergence $\hat{h}_{it} \to 1$ for $t \to \infty$. Under the same conditions

$$\hat{s}_t^2 = \frac{1}{N}\sum_{i=1}^{N}\left(h_{it} - 1\right)^2 \to 0, \text{ as } t \to \infty. \qquad (6.5)$$

Under growth divergence, i.e heterogeneity of l_{it} as $t \to \infty$, the cross-sectional variance of $\hat{h}_{it}$ will no decline to zero.

In the following figure we present for the sample of 101 countries, the sample variance computed by using (6.5) and its approximate 95% confidence band for each case computed by bootstrapping the estimate $\hat{g}_{it}$.

Fig. 5 Cross-sectional variance
Common Component, HP filter



There is no evidence of steady decline in $\hat{s}_t^2$ over time, as should be in case of convergence. We conclude for per capita income divergence during the period 1960-1998 and for the sample of 101 countries included in the analysis.

7. The least square dummy variable strategy.

In this section, we briefly review the statistical model originally proposed by Stockman (1988) and Costello (1993) to analyze the productivity dynamics in the OCDE countries, which was used by Bayomi and Prasad (1997) to study currency area properties in Europe, by Marimon and Zilibotti (1998) to study European employment dynamics, and by Loayza et al. (2001) to analyze common real value-added patterns for Latin America, East Asia and European countries.

The model has been applied to labour productivity variable for a sample of 73 countries. In brief, the data for output comes from the World Bank and are given by the gross value added in the agricultural, industry and service sectors during the period 1974 - 1999. The variables, originally expressed in constant local currency units, were converted to 1985 international dollars by using the corresponding purchase power parity index reported in Penn World Table (Mark 5.6a). Data for labour comes from the ILO database and reflect the economically active population in the agricultural, industrial and service sectors. The availability of data determined which countries were included in the study.

Formally, we assume that the labour productivity growth rate can be decomposed in aggregate international effects associated with the business cycle, sector-specific effects connected, for example, to sectoral technological trends, and country specific factors which can have either an aggregate and/or industry specific nature. If this is the case, in each period *t* the labour productivity growth rate in country *j* and sector *i* can be decomposed as the sum of the following components:

$$\Delta p_{ijt} = h_i + b_t + f_{it} + m_{ij} + g_{jt} + u_{ijt} \qquad (7.1)$$

for sector i=1, 2,..., I, country j=1, 2,..., N and time t=1, 2,..., T, where

- Δp_{ijt} is the labour productivity growth rate of sector *i*, in country *j* at time *t*.
- h_i is a time-invariant component specific for sector *i* but common to all countries. Its function is to capture the mean growth rate across countries in sector *i* and represent the international trend of labour productivity in sectoral growth rates.
- m_{ij} is a time-invariant component that identifies deviations across countries from h_i. These differences may be connected to different initial conditions, for example.
- b_t is a time effect common to all *N* countries and sectors *I*. The main aim of this term is to capture the international business cycle that influences all countries and sectors.
- f_{it} captures deviations across time from h_i, and deviations across sectors from b_t; the function is to capture differences in cyclical behaviour of a specific sector in a country.

- g_{jt} captures country-specific deviations from b_t; for example, transitory national performance with respect to the international business cycle resulting from national economic policies.
- u_{ijt} is an error term orthogonal for all effects.

The previous model is not identified because some combinations of the dummy variables are perfectly collinear. For example, it is simple to see that the matrix $\left[h(i)\sim m(i,n)\right]$ has I linear dependent columns and the matrix $\left[f(i,t)\sim g(n,t)\right]$ has $\left(N+I+T-1\right)$ linear dependent columns. Finally, the matrix of dummies $b(t)$ can be obtained as combination of the previous $\left[f(i,t)\sim g(n,t)\right]$ matrix, i.e T columns are linear dependent. In conclusion $\left(2T+2I+N-1\right)$ columns are linear dependent. Thus the model cannot be estimated without introducing at least $\left(2T+2I+N-1\right)$ restrictions. In this case, we follow the methodology provided by Bayomi and Prasad (1997) and Marinon and Zilibotti (1998), who assume that all the different effects highlighted in equation (7.1) are orthogonal. Unlike Stockman (1988) and Costello (1993), who choose a specific country and time period as the reference point, assuming orthogonality between all the elements in (7.1) implies taking as a reference point not a single country, sector or year, but the respective sample averages.

More specifically we assume that

- $\sum_{j=1}^{N} m_{ij} = 0,\ i=1,\ldots,I,$
- $\sum_{i=1}^{I} f_{it} = 0,\ t=1,\ldots,T,$
- $\sum_{t=1}^{T} f_{it} = 0,\ i=1,\ldots,I,$
- $\sum_{t=1}^{T} g_{jt} = 0,\ j=1,\ldots,N,$
- $\sum_{j=1}^{N} g_{jt} = 0,\ t=1,\ldots,T,$
- $\sum_{t=1}^{T} b_t = 0$

which give a set of $2T + 2I + N + 1$ restrictions, of which all but two are independent. Thus, given that $\left(2T+2I+N-1\right)$ columns of dummies are linear dependent, the model is exactly identified.

A way to analyse the importance of common sectoral effects when influencing long-term labour productivity is to set all countries-specific components (i.e. $m_{ij} = g_{jt} = u_{ijt} = 0$) to zero and define a new labour productivity variable, which is given by the following expression:

$$\Delta p_{it} = h_i + b_t + f_{it}. \qquad (7.2)$$

Note that in this case labour productivity does not depend on country effects, and that its growth is connected to common factors which influence sectors in all countries by the same amount. In synthesis, expression (2) shows the sector labour productivity growth rate that countries would have experienced in the absence of any "country idiosyncratic" effect.

We are now ready to present the regression results. The model described in (7.1) was estimated by using a dummy variable regression method for the panel of labour productivity growth rates described in the previous section.

For reasons of brevity we do not report all the estimates, but it is worth mentioning that more than 50% of the labour productivity growth rate variance is explained by the model. The coefficient of determination is R^2 =0.51.

In table 2, we report the analysis of long-run and short-run variations of sectoral labour productivity growth rates. As is usual when reporting these values, both components have been normalized to add up to 100 percent. As can be seen from table 2, more than 86% of the total variation in long-run trends for the total sample of countries are explained by the country specific factors m_{ij}.

The same picture emerges when short-run variations are examined. The country-specific effects g_{jt} plays the major role. International business cycle b_t and sectoral factors f_{it} have a lesser effect. In synthesis, labour productivity growth rates largely depend on country-specific effects.

Table 2. Analysis of long and short-run variations

Total Sample of Countries			
Components	Analysis of long-run variations		
	% explained	Variance	Correlation
p_L	100.00	0.0006	1.00
h_i	13.75	0.0001	0.37
m_{ij}	86.25	0.0005	0.93
	Analysis of short-run variations		
	% explained	Variance	Correlation
p_S	100.00	0.0024	1.00
b_t	5.17	0.0001	0.23
f_{it}	1.32	0.0000	0.10
g_{jt}	93.51	0.0023	0.97

Sources : author's calculation based on World Bank and ILO dataset

The term h_i, that accounts for sector-specific effects, which are, by definition, country and time independent, explains the remaining variance.

At this point we use equation (7.2) to estimate for each country what we label "common trend of labour productivity", i.e. an estimate of the sectoral labour productivity trend that each countries eventually would have shared in the absence of idiosyncratic shocks.

Specifically, for each country the level of labour productivity has been obtained by taking as initial condition the actual level of labour productivity in 1974 and applying to it, up to final year, the sequence of growth factors highlighted in equation (7.2).

We define the following variances

$$s_{it}^{2} = \frac{1}{N}\sum_{j=1}^{N}\left(\log y_{ijt} - \log \overline{y}_{ijt}\right)^{2}$$

for each sector $i = 1,...,I$ and time $t = 1,...,T$. Naturally we expect that under convergence the variances will not grow over time.

Fig. 7 Cross-sectional variance
LSDV approach

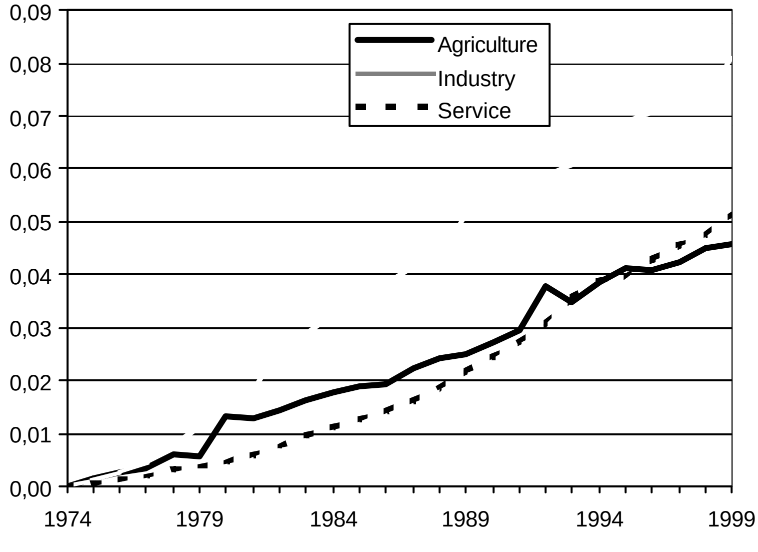


Figure 7 seems to deny convergence. All the sectors show increasing variances during the period 1974-1999. Thus we conclude that also in this case as well as in the previous cases, instead of convergence, divergence seems to characterize world-wide path of per capita income and labour productivity.

Conclusions

In this note we have analysed the issue of convergence of per capita income and labour productivity for a large cross section of countries and sectors during the last three decades. As in previous studies, the analysis shows that convergence strongly depends on similar growth rates of technological progress across countries. If countries do not experience convergence of technological progress, at least in the long run, their per capita incomes or labour productivities will diverge. This consideration probably explains why when studying a large and heterogeneous set of countries, researchers usually find no sign of convergence while when analysing a more homogeneous panel of countries, some report evidence for convergence. In our note technological progress is an exogenous variable and, as is the case in many other works, its path continues to not be analysed and this naturally limits our knowledge of economic growth and convergence.

However, in the note we show that when the hypothesis of common technological progress has been introduced countries can still diverge for a long period of time if there is divergence in their transitional growth paths.

To investigate these issues empirically we introduce some instruments mainly derived from recent literature on common trends. We show that when a country or a panel of countries experiences similar rates of technological progress, the analysis of common trends can throw light on their transition path.

We use three different methodologies, two based on panel estimation methods and one on the well known Hodrick and Preston (1997) trend estimation method. The panel estimation method is based on Bai and Ng's (2003) common principal components. We show that, for our sample of countries, differences of log per capita income is strongly characterized by a common stochastic trend, i.e. countries do not converge in the long-run. We obtain the same results when using the least squares dummy panel estimation method proposed by Marimon and Zilibotti (1998). In this case labour productivity computed for a set of 73 countries and for the three sectors (agriculture, industry and services) diverges from the common trend.

Finally we compute Hodrick and Prescott (1997) to estimate the trend component of log per capita income for each country in the sample. When computing the variance of the ratio between the trend and the mean panel trend for each country, we show that the statistic diverges, highlighting once more that countries do not converge.

Many other methods may be used to estimate common trends. Basically two important ones have been recently proposed. One is based on spectral analysis (Forni *et al*. 2000), and the other on the Kalman Filter (see for example Engle and Watson, 1981). The use of these methods leaves space for further research on the theme of convergence.

Reference

Andrew D.W.K (1993). Exactly Median-Unbiased Estimation of First Order Autoregressive / Unit Root Models. *Econometrica*, 61, 139-165.

Bai, J., Ng, S. (2002). Determining the Number of Factors in Approximate Factor Models. *Econometrica*, 70:1, 191-221.

Bai, J., Ng, S. (2003). A PANIC Attack on Unit Roots and Cointegration, mimeo, Boston College.

Barro, R.J., and Sala-i-Martin, X. (1995). *Economic Growth*. New-York: McGraw-Hill.

Bayomi, T., Prasad, E.(1997). Currency Unions, Economic Fluctuations, and Adjustment: Some New Empirical Evidence. *IMF Staff Papers*, 44:1, 37-58.

Breitung, J., Meyer, W. (1991). Testing for Unit Roots in Panel Data: are Wages on Different Bargaining Levels Cointegrated? Institute für Wirtschaftsforschung Working Paper, June.

Campbell, J., Perron, P. (1991). Pitfalls and Opportunities: What Macroeconomists Should Know about Unit Roots. *NBER Macroeconomics Annual*, MIT Press.

Costello, D. (1993). A Cross-Country, Cross-Industry Comparison of the Behavior of the Solow Residuals. *Journal of Political Economy*, 101, 207-222.

Dickey, D.A., Fuller, W. A. (1981). Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root. *Econometrica*, 49: 1057-1072.

Diebold, F.X., Nerlove, M. (1990). Unit Roots in Economic Time Series: A Selective Survey. In *Advances in Econometrics: Cointegration, Spurious Regressions, and Unit Roots*, edited by T.B Fomby and G.F. Rhodes. Greenwich, CT:JAI Press, 3-70.

Engle, R.F., and Watson M. (1981). A One-factor Multivariate Time-Series Model of Metropolitan Wage Rates. *Journal of American Statistical Association*, 76: 774-781

Forni, M., Hallin, M., Lippi M., Reichlin L. (2000). The Generalized Factor Model: identification and estimation. *The Review of Economics and Statistics,* 82, 540-554.

Heston A., Summers R. and Aten B. (2002) Penn World Table Version 6.1, *Center for International Comparisons at the University of Pennsylvania (CICUP)*, October.

Hodrick, R.J and Prescott E.C. (1997) Postwar U.S. Business Cycles: An Empirical Investigation. Journal of Money Credit and Banking, 29, 1-16.

Kwiatlowski, D., Phillips, P.C.B., Schmidt, P., and Shin Y. (1992). Testing the Null Hypothesis of Stationary against the Alternative of a Unit Root, *Journal of Econometrics*, 54, 159-178.

Im, K.S., Pesaran, M.H., Shin, Y. (1997). Testing for Unit Roots in Heterogeneous Panels, mimeo, Department of Applied Economics, University of Cambridge.

Levin, A., Lin, C.F. (1992). Unit Root Tests in Panel Data: Asymptotic and Finite-Sample Properties. Discussion Paper Series 92-23, Department of Economics, University of San Diego.

Levin, A., Lin, C.F. (1993). Unit Root Tests in Panel Data: New Results. Discussion Paper Series 93-56, Department of Economics, University of San Diego.

Loayza, N., Lopez, H., Ubide, A. (2001). Comovements and Sectoral Interdependence: Evidence for Latin America, East Asia, and Europe. *IMF Staff Papers*, 48:2, 367:396.

Maddala, G.S., Wu, S. (1999), A Comparative Study of Unit Root Tests with Panel Data and a New Simple Test, *Oxford Bulletin of Economics and Statistics*, 61, 631-652.

Marimon, R., Zilibotti, F. (1998). Actual Versus Virtual Unemployment in Europe: Is Spain Different?, *European Economic Review*, 42, 123:153.

Moon, H. R., Perron, P. (2002). Testing for Unit Root in Panels with Dynamic Factors. Reasearch Papers Series, University of Southern California Center for Law, Economics & Organization, n. C01-26.

Phillips, P.C.B., Perron, P. (1988). Testing for a Unit Root in Time Series Regression. *Biometrika*, 75: 335-346.

Phillips, P.C.B., Sul, D. (2002). Dynamic Panel Estimation and Homogeneity Testing under Cross-Section Dependence, Cowles Foundation Discussion Paper n.1362.

Phillips, P.C.B and Sul, D. (2003) The Elusive Empirical Shadow of Growth Convergence. Cowles Foundation Discussion Paper n°. 1398.

Quah, D. (1990). International Patterns of Growth: I. Persistence in Cross-Country Disparities. MIT Working paper, January.

Quah, D. (1994). Exploiting Cross-Section Variations for the Unit Root Inference in Dynamic Data. *Economics Letters*, 44: 9-19.

Ravn, M.O. and Uhlig H. (2002) On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations. *Review of Economics and Statistics*, 84, 371-376.

Stockman, A. (1988). Sectoral and National Aggregate Disturbances to Industrial Output in Seven European Countries, *Journal of Monetary Economics*, 21, 387-409.